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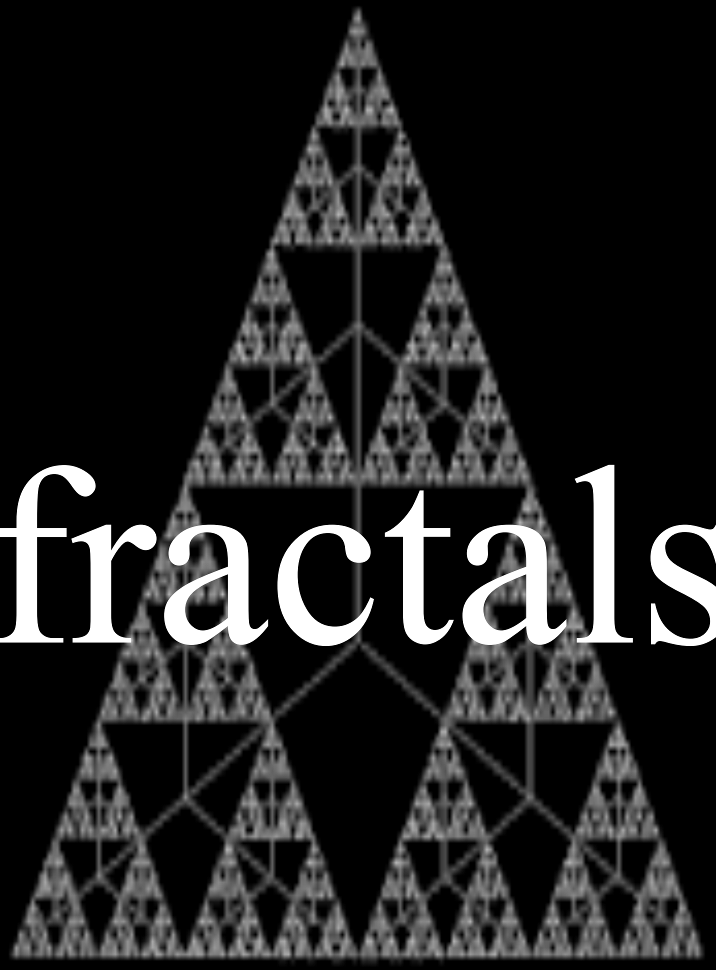

fractals

chaos theory

Mattia Quatrale

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"Mathematicians believed that prediction was just a function of keeping track of things. If you knew enough, you could predict anything… Chaos theory throws it right out the window because in fact there are great categories of phenomena that are inherently unpredictable."
Michael Crichton

What is entropy?

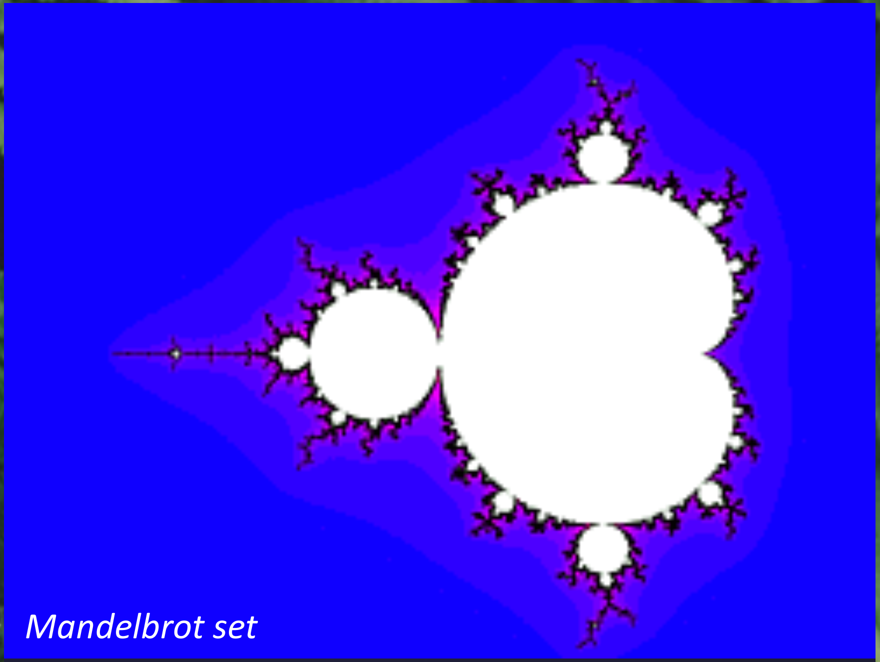
The word entropy broadly indicates the degree of disorder or uncertainty of a system.

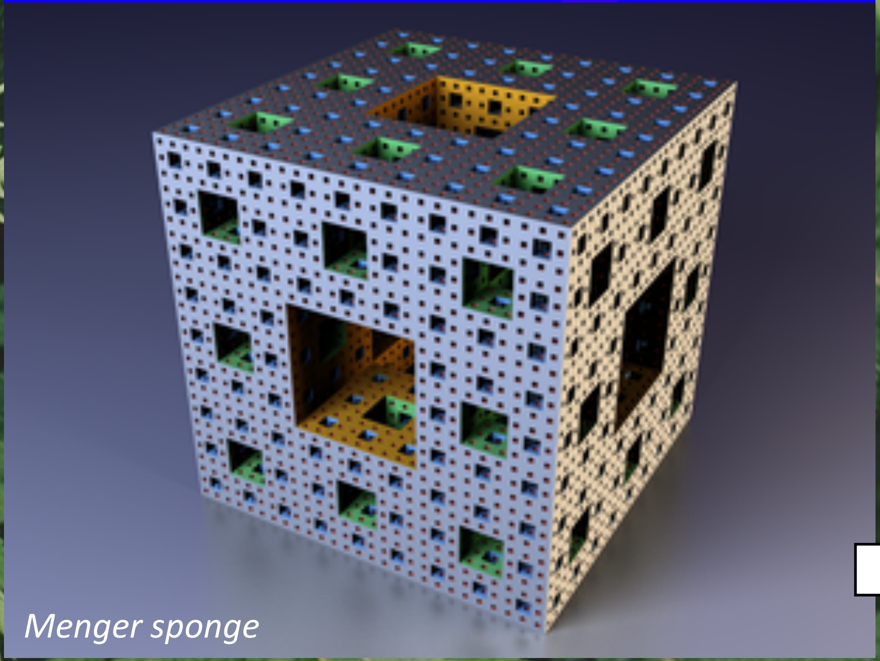
Transformations always rise the total level of entropy of the universe so entropy will rise more and more up to a maximum level where there will be no more heat exchanges.

Definition:
In thermodynamics entropy is a measure of the unavailable energy in a closed thermodynamic system that is also usually considered to be a measure of the system's disorder, that is a property of the system's state, and that varies directly with any reversible change in heat in the system and inversely with the temperature of the system.
Formula:
$S = k_B \ln \Omega (assuming\ equiparable\ micorstates)$

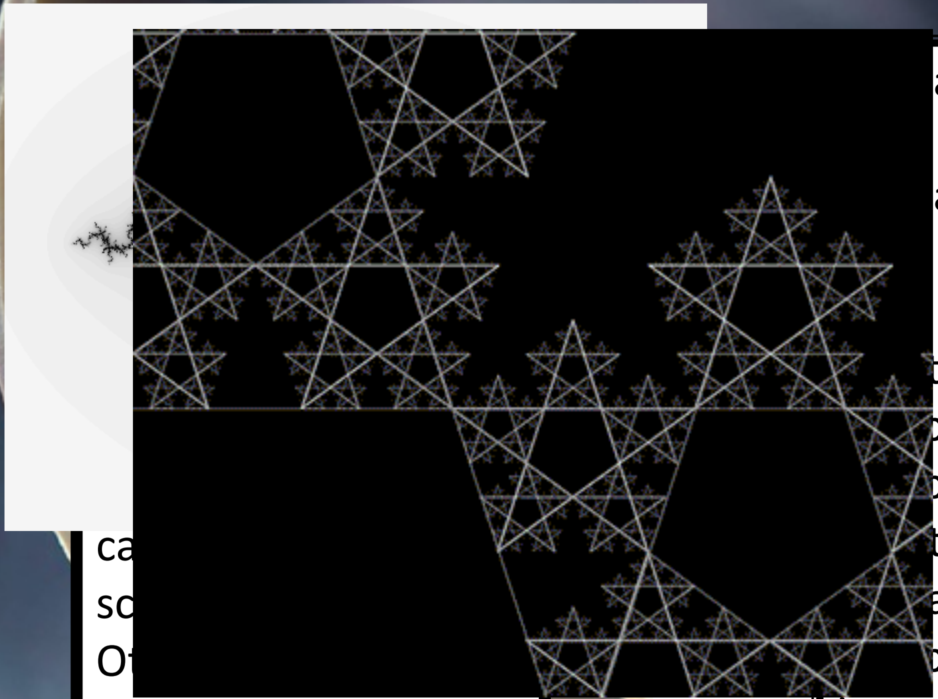
What are fractals?

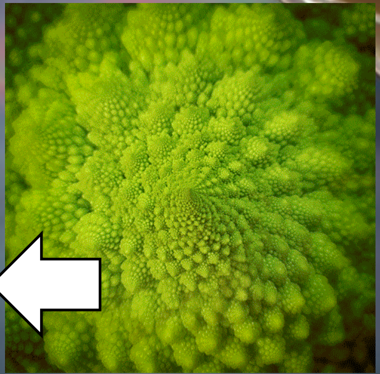
In mathematics, a fractal is a detailed, recursive, and infinitely self-similar mathematical set. Fractals are encountered ubiquitously in nature due to their tendency to appear nearly the same at different levels. The pictures exhibit similar patterns at increasingly small scales, also known as expanding symmetry or unfolding symmetry; If this replication is exactly the same at every scale, as in the Menger sponge, it is called a self-similar pattern.


Mandelbrot set


Menger sponge

Fractals examples





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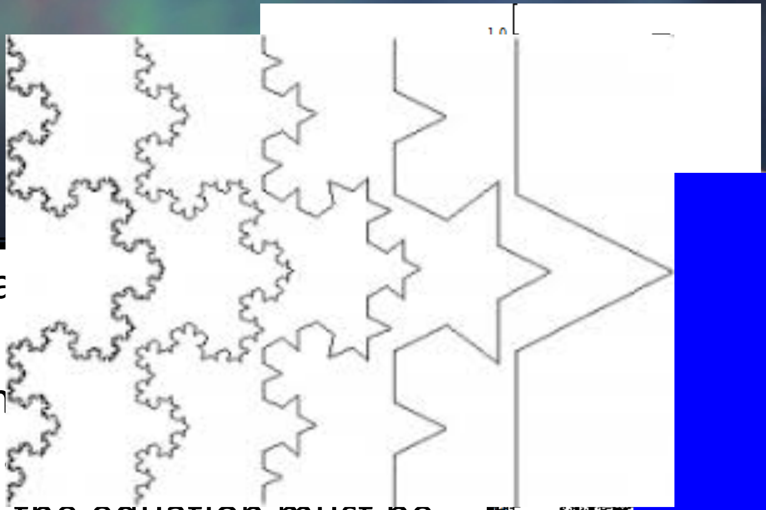

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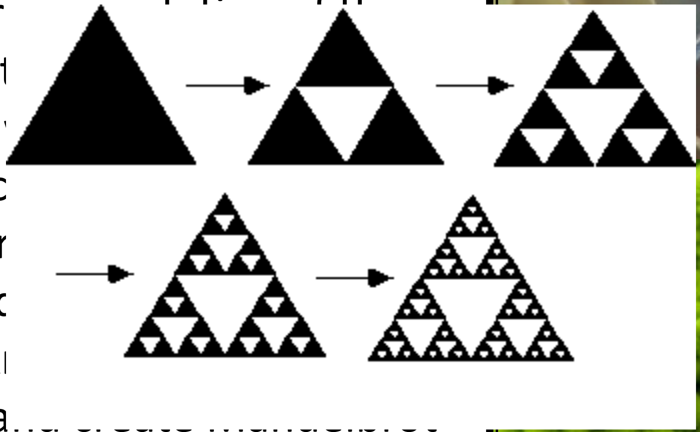
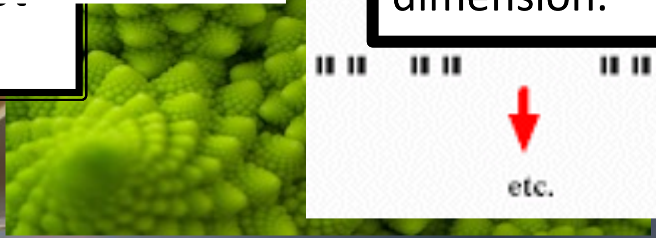
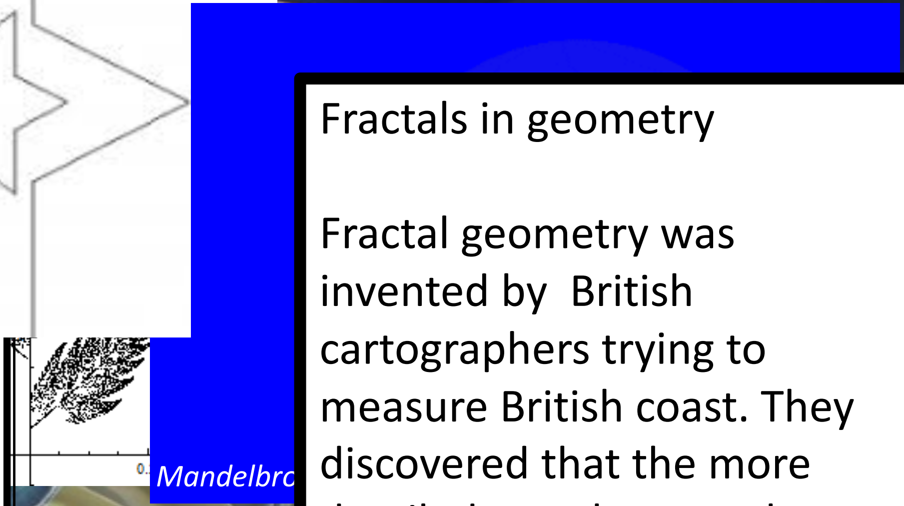
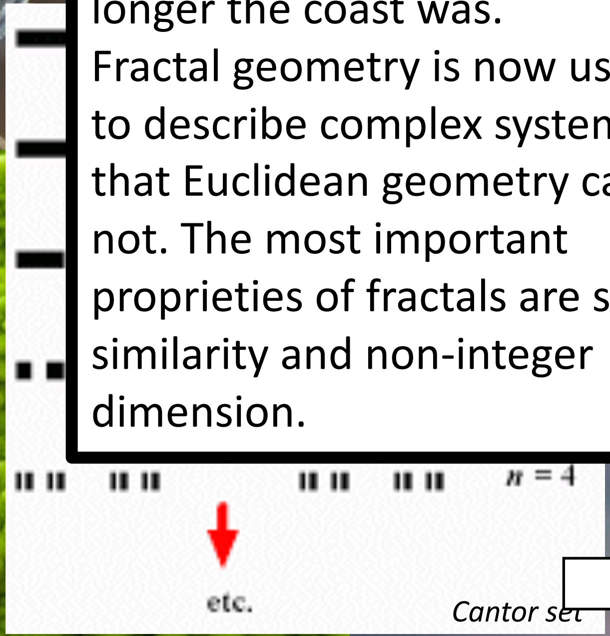








Fractals in geometry

Fractal geometry was invented by British cartographers trying to measure British coast. They discovered that the more detailed was the map the longer the coast was. Fractal geometry is now used to describe complex systems that Euclidean geometry can not. The most important proprieties of fractals are self-similarity and non-integer dimension.

Chaos theory

"In all chaos there is a cosmos, in all disorder a secret order." Carl jung

Chaos theory is a branch of mathematics focusing on the behavior of dynamical systems that are highly sensitive to initial conditions. "Chaos" is an interdisciplinary theory stating that within the apparent randomness of chaotic complex systems, there are underlying patterns, constant feedback loops, repetition, self-similarity, fractals, self-organization, and reliance on programming at the initial point known as sensitive dependence on initial conditions. Chaos theory basically states that a system where no randomness is involved in generating future states in the system can still be unpredictable. According to chaos theory we can't predict which state the system will be in at time t, but we can (apparently) predict the general shape the states of the system move through over time.

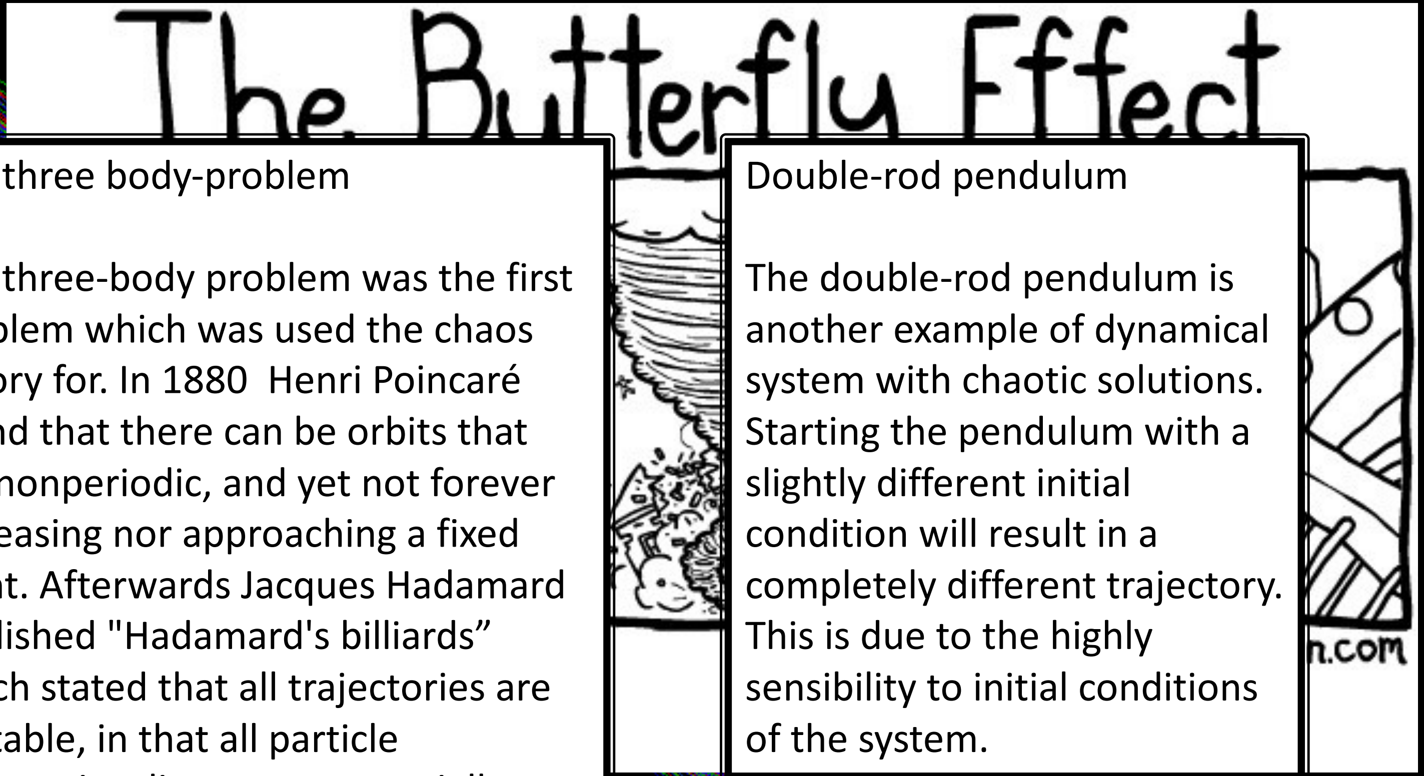
Some chaos theory examples

Butterfly effect

consequences.
It also involves the climate theme which is something chaotic scientists have to deal with frequently.

The three body-problem

The three-body problem was the first problem which was used the chaos theory for. In 1880 Henri Poincaré found that there can be orbits that are nonperiodic, and yet not forever increasing nor approaching a fixed point. Afterwards Jacques Hadamard published "Hadamard's billiards” which stated that all trajectories are unstable, in that all particle trajectories diverge exponentially from one another.



Double-rod pendulum

The double-rod pendulum is another example of dynamical system with chaotic solutions. Starting the pendulum with a slightly different initial condition will result in a completely different trajectory. This is due to the highly sensibility to initial conditions of the system.

chaos theory, entropy and fractals

According to chaos theory we can't predict specifically the state of a system at a precise time, but we know it will follow and orbit-like scheme. This scheme is similar to the Earth-Moon one and just like it it has an attractor (which is the Earth in the example). Attractors are generally points or curves that the system tries to conform with. In the case of a chaotic system we have a different kind of attractors that are called "strange attractors" or "fractals" which could be an infinite set of unconnected points (e.g. a Cantor dust), or a smooth curve with mathematical discontinuities, or a curve that is fully connected but discontinuous everywhere for example.

Chaos theory states that even if a system looks random it follows an orbit. So we can predict the shape of it even if we can't know its pattern in advance. Similarly entropy is a state function, which means that it depends only on the initial and final results and not on the patterns, at this point it is possible to understand how chaos theory and entropy are related.

"We live in a rainbow of chaos." Paul Cezanne

Sitography

Entropy definition: https://www.merriam-webster.com/dictionary/entropy

Fractal geometry:
http://www.fractal.org/Bewustzijns-Besturings-Model/Fractals-Useful-Beauty.htm
https://sites.google.com/site/fractalgeometrykk/fractals/geometric-fractals

Fractals in algebra:
https://sites.google.com/site/fractalgeometrykk/fractals/algebraic-fractals

Fractals information and about nature:
https://en.wikipedia.org/wiki/Fractal

Chaos theory:
https://en.wikipedia.org/wiki/Chaos_theory
https://fractalfoundation.org/resources/what-is-chaos-theory/

"As far as the laws of mathematics refer to reality, they are not certain, and as far as they are certain, they do not refer to reality." Albert Einstein